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POLYNOMIAL AND POWER FUNCTIONS FOR GLACIAL VALLEY CROSS-SECTION MORPHOLOGY

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ABSTRACT

An empirical evaluation of glacial trough cross-section shape is performed on seven vertical cross-sections in three Sierra Nevada valleys glaciated during the late Quaternary. Power and second-order polynomial functions are fitted by statistical regression. Power functions are very sensitive to subtle valley-bottom topographic features and require precise specification of the valley-bottom-centre location. This dependency is problematic given under-representation of valley bottoms by conventional contour-sampling methods, and the common alteration of valley-bottom morphology by non-glacial processes. Power function exponents vary greatly in response to these and other non-genetic factors and are not found to be reliable indicators of overall valley morphology.

Second-order polynomials express overall valley shape in a single robust function. They are applied to both bedrock- and sediment-floored glacial valleys with negligible statistical bias except where side-slopes are stepped or convex-upward or where valley form is asymmetrical. They can describe alluviated or severely eroded valleys, and can objectively identify individual components of polymorphic valleys, because valley bottom and centre locations need not be specified. Mathematical expressions of parameters useful for geomorphic measurements and glaciological modelling are analytically derived from the polynomials as functions of the three polynomial coefficients. These parameter equations provide estimates of valley side-slopes, mean and maximum depth, midpoint location, width, area, boundary length, form ratio and symmetry.

KEY WORDS glacial geomorphology; trough form; slope profiles; morphometry; Sierra Nevada California

INTRODUCTION

It is commonly believed that glacial trough cross-section morphology can be described as a parabolic curve. The early history of this notion is explored by Harbor (1989) who identifies McGee (1883) as both a proponent of the concept and a pioneer in the examination of process–form relationships. A common quest in glacial geomorphology over the last quarter century has been the derivation of simple mathematical expressions of valley topographic form. Such expressions can provide tools for geomorphic measurement, data description and reduction, generation of model parameters, and for testing and understanding theories of process–form relationships. Specifically, several attempts have been made to express the two-dimensional topographic form of glaciated valleys as a function of lateral distance across the valley. These studies have been motivated by attempts to differentiate glacial from fluvial valley forms, to quantify the degree or intensity of glacial erosion (Doornkamp and King, 1971; King, 1974; Hirano and Aniya, 1988), and to develop numerical models of glacial trough evolution (Harbor *et al.*, 1988; Harbor, 1992).

Because of the great variability in glacial valley morphology, no single, simple function has been identified or is likely to emerge that can adequately describe all glacial valley cross-sections. Doornkamp and King (1971) applied 16 different univariate mathematical functions to three glacial valleys. Based on correlation coefficients, squares of both the independent and dependent variables (lateral and vertical distance, respectively) provided the best least-squares fit in two cases but did poorly in the third case. Semi-log expressions with log elevation yielded high coefficients of determination in all three cases. Power functions (log–log)

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provided satisfactory regressions in two of the three cases, but were inappropriate in the third case. These variations in the best-fit model suggest that no single model is best in all cases and that the conventional power model is not necessarily superior to other alternatives. This paper tests the power function model and an alternative quadratic model on seven Sierra Nevada valley transects that were glaciated at least twice during the late Quaternary to ice depths in excess of 300 m.

Power functions

Power functions are conventionally derived empirically through the use of a monomial expressing elevation above the trough floor as a power of lateral distance from the trough-bottom centre (Svensson, 1959; Graf, 1970; Doornkamp and King, 1971; King, 1974; Embleton and King, 1975; Harbor and Wheeler, 1992):

$$H = dX^f \tag{1}$$

where H is height above the lowest point in the section, X is lateral distance from the trough centre, d is a coefficient, and the exponent (f) is the logarithmic slope of the function. These functions express the form of a half-valley, so it is not uncommon to average data from both sides to develop a single ideal half-valley curve (Svensson, 1959; Graf, 1970). Since the resultant curves are rarely true parabolas with an exponent of 2·0, these models are referred to in this report as 'power functions.' Svensson (1959) claimed that this form yields the best approximation of glacial valley cross-sections.

Several studies have suggested that valley morphology progressively approaches a true parabolic form with increasing extent of glacial erosion, and that stage of valley evolution can be detected through increases in the power function exponent toward 2·0 (Svensson, 1959; King, 1974; *cf.* Graf, 1970; Hirano and Aniya, 1988). The premise is that non-glacial valleys have relatively linear sides with exponents close to one, but that glaciation ultimately widens valley bottoms into a parabolic shape. For example, King (1974) interpreted a valley with an exponent close to 2·0 as having been actively glaciated, while two other valleys with exponents of 1·68 and 1·75 were occupied by diffluent glaciers and were less actively glaciated. Graf (1970) pointed out that power function exponents can provide a complete description of valley form only by the additional specification of the form ratio (depth/width). He found, in samples from 61 Beartooth Mountain valleys, that higher-order valleys with greater ice discharges tended to be deeper, narrower and closer to parabolic than lower-order valleys, but that exponents tended more towards 1·6 than 2·0. The hypothesis of progressive, systematic increases in power function exponents has been criticized by Harbor (1990) since it assumes that glacial erosional forms tend towards maximum efficiency or minimum friction, neither of which has been demonstrated.

Power functions have also been applied to glaciated valleys in an attempt to identify valley bottoms buried in alluvium or where glaciers remain. The difficulty with this application is that valley-bottom horizontal and vertical locations must be specified to define the coordinate system before functions can be derived (Wheeler, 1984). Aniya and Welch (1981) attempted to resolve this dilemma by calculating power functions using various valley-bottom depths and iteratively converging on the origins for the function that yielded the highest explained variance. Valley-bottom coordinates are varied in this study to evaluate the feasibility of this procedure.

Polynomial functions

An alternative model of glacial valley cross-section form is a quadratic equation in the form of a second-order power series:

$$E = a + bS + cS^2 \tag{2}$$

where E is elevation above a datum such as sea level, S is lateral distance from the reference station, and a, b and c are coefficients (Wheeler, 1984; Augustinus, 1992a). Second-order polynomials provide robust parabolic expressions of valley form largely free of the data requirements and limitations of power functions, although their flexibility is limited by the fixed exponent. By fitting both sides of the valley at once, these equations provide a measure not only of hillslope form but also of overall two-dimensional valley shape.

Svensson (1958) dismissed the polynomial equation as merely a means for locating the coordinates for power functions, which he favoured owing to their greater versatility. Girard (1976) and Svensson (1958) derived polynomial functions using the valley centre point to define the horizontal coordinate system axis. The present study utilizes a datum at the upper valley wall where lateral moraines, trim lines or erratics can provide a clear limit to the glacial extent based on field evidence. This datum is referenced to coordinates of zero in the horizontal direction and elevation above sea level in the vertical dimension. This axis definition is optimal where mapping has located ice elevations on valley walls, or where trough bottoms have been altered. It also permits the use of analytical depth and shape parameters presented at the end of the paper.

THE STUDY SECTIONS

Seven valley transects were sampled from three valleys in the northwestern Sierra Nevada of California. Six of the transects, three from Bear Valley and three from South Yuba Canyon (the Yuba Gorge), are located within a small area covered by the Blue Canyon 1:24,000 U.S. Geological Survey quadrangle (Figure 1). These six sections were selected to analyse the morphological dichotomy between two neighbouring glacial troughs that are part of another study testing a hypothesis of Quaternary glacial drainage diversion (James,

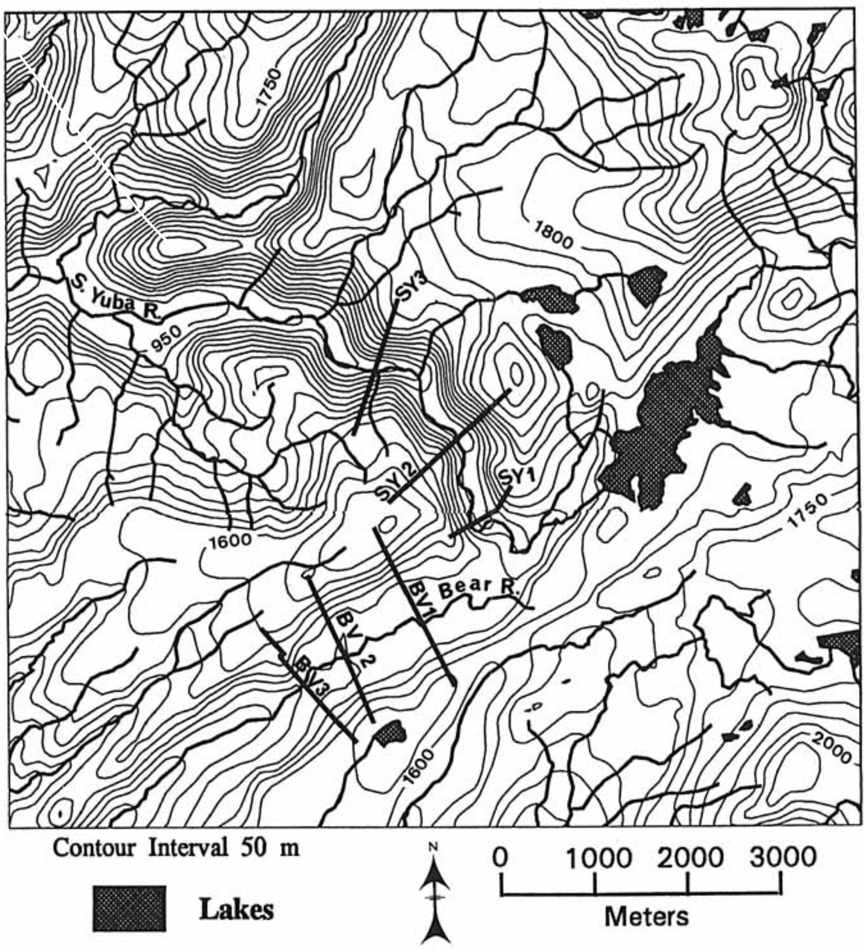


Figure 1. Locations of the transects in Bear Valley and Yuba Gorge. The South Yuba River turns abruptly 110° at the site of a glacial diversion into Yuba Gorge. Contours generated from 1:250,000 digital elevation model; hydrography from Tiger file GIS coverages

in press). The Tenaya Canyon transect is located about 200 km to the south-southeast in Yosemite Valley. The valleys at most of the transects are eroded into relatively resistant granitic or metamorphic rocks (Table I), but the two upper Bear Valley transects cross an exhumed palaeovalley with walls of incompetent epiclastic fill, floored by an unknown depth of outwash.

The upper Bear Valley is a broad, straight trough about 2·5 km wide and 250 m deep that extends about 2 km longitudinally between metamorphic outcrops (Figure 2). Two valley sections (BV1 and BV2) are located in this broad portion of the valley. Metamorphic benches between elevations of 1370 and 1500 and 1600 m at the lower end of Bear Valley represent eroded spurs normal to the Bear Valley axis. The ancestral Yuba channel passed through the northwest wall of Bear Valley between the benches in a deep, wide palaeovalley that was later filled with more than 250 m of andesitic lahars (Lindgren, 1900, 1911; James, in press). Bear Valley is broad where the palaeovalley has been exhumed and narrows across the bench at transect BV3. Bench surfaces were deeply glaciated at least twice during the late Quaternary, as is evidenced by moraines high on valley walls (James, in press), so differences in trough morphology between BV3 and the other Bear Valley sections are interpreted as responses to differences in lithology and valley gradient.

Yuba Gorge is a deep, narrow, sinuous gorge cut into metamorphic and granitic rock (Figure 3). Evidence of glaciation includes moraine ridges and erratics on the upper valley rim and polish and striae within the gorge (James, in press). The gorge held an outlet glacier and apparently carried most of the subglacial meltwater from the west side of the ice field during the last glacial maximum. The gorge has very steep longitudinal slopes reaching a maximum gradient greater than 8 per cent averaged over 0·75 km across rock bar surfaces below the junction with Bear Valley.

Tenaya Canyon is higher than the other valleys and was also occupied by late glacial ice (Matthes, 1930). The transect is across a deep, narrow gorge eroded into granitic rock 6 km upvalley from the main Yosemite Valley and 4 km up from Half Dome. Tenaya is included to represent a deep, narrow valley with a long-studied glacial history.

The effects of bedrock and valley gradient are not explicitly examined in this study, but appear to be important variables determining valley shape. The five narrow valley cross-sections are all in igneous or metamorphic rocks and the four deepest of these sections (South Yuba and Tenaya sections) are also associated with steep gradients. The two wide Bear Valley sections are developed in incompetent rock in a valley of low gradient. This relationship is in agreement with the observations of Embleton and King (1975, p. 253) that V-shaped glaciated valleys are associated with steeper gradients than U-shaped glacial valleys. It is also in agreement with the findings of Augustinus (1992a, b) that New Zealand Fiordland valleys eroded in lithologies of high rock mass strength (RMS) have narrower valleys than those with low RMS in fractured zones.

Table I. Valley transect characteristics

		Rock type		
Location	ID	Upper	Lower	Floor
Bear Valley 1	BV1	Epiclastic	Epiclastic	Alluvium
Bear Valley 2	BV2	Epiclastic	Epiclastic	Alluvium
Bear Valley 3	BV3	Epiclastic	Metamorphic	Metamorphic
South Yuba 1	SY1	Metamorphic	Metamorphic	Metamorphic
South Yuba 2	SY2	Granitic	Granitic	Granitic
South Yuba 3	SY3	Granitic	Granitic	Granitic
Tenaya Canyon	TEN	Granitic	Granitic	Granitic

Epiclastic = andesitic lahars; primarily poorly sorted, unconsolidated breccia
Metamorphic = weakly metamorphosed, strongly deformed, Palaeozoic sandstone and greywacke (quartzite); the Lang-Halsted unit of the Shoofly Complex. Almost vertical bedding strikes approximately north–south
Granitic = felsic, Devonian granitics of Bowman Lake pluton, variously mapped as a true granite and trondhjemite

Figure 2. View south across Bear Valley at Transect 2 which begins at arrow and crosses at dashed line above a low recessional moraine. Transect 1, out of view to the left, also has a wide, flat alluvial floor with widely spaced valley walls composed of layered epiclastics

Figure 3. View east up Yuba Gorge across SY3 showing straight valley walls and steep longitudinal gradient eroded in granitic batholith; contact with metamorphic rock is upvalley, out of view

The roles of lithology and gradient in glacial trough deepening pose a limit on the ability to assess glacial processes from trough cross-section morphology alone.

SAMPLING PROCEDURES

Valley cross-sections were sampled from 1:24,000 topographic maps with 12·2 m (40 ft) contour intervals, except for Tenaya Canyon which was sampled from Matthes' (1930) map with 15·2 m (50 ft) contour intervals. Map distance along each transect was measured under magnification to the nearest 0·1 mm at each point where a contour line crosses the transect; each station-elevation data pair was entered into a computer spreadsheet. This 'contour-crossing' sample method, commonly employed to generate valley cross-sections (e.g. King, 1974), is biased towards steep slopes and under-represents flat areas, such as valley centres, where there are few contour lines. Alternative sampling methods that could eliminate sample bias (e.g. field topographic surveys or photogrammetry) or improve resolution (e.g. digital elevation models or interpolations of points between contour lines) may not be warranted.

Numerous problems are associated with the use of valley-bottom data. First, log–log least-squares solutions used to derive power functions are biased towards small distances and elevations. For this reason, Harbor and Wheeler (1992) advocate measuring valley-bottom points as accurately as possible and emphasizing these locations in sampling. The contour sample method is biased towards steep slopes, however, so the points most influential to curve-fitting are poorly represented by samples. The importance of this sample bias was tested in the two upper Bear Valley transects using data augmented by interpolated valley-bottom observations.

To characterize glacial valley form with power functions, the appropriate surface to sample is the bedrock valley floor free of post-glacial sediment or erosion. Alluvial surfaces should be avoided because they can cause increases in power function exponents (Graf, 1970; Aniya and Welch, 1981; Harbor and Wheeler, 1992). Unfortunately, the power function method is not simply influenced by valley-bottom data, but requires the valley-bottom centre to be precisely located horizontally and vertically in order to define the origin of the coordinate system (Wheeler, 1984). The need to specify precisely the origin and valley-bottom data presents a serious dilemma in areas of post-glacial cut or fill, which are common.

Given variations in methods of data collection and coordinate system definition, this study performs a sensitivity analysis on curve-fitting methods using datasets with three different degrees of valley-bottom representation: all contour data alone, contour data augmented in valley bottoms (power functions only), and all valley-bottom data omitted. Power function coordinate system reference points are also varied to examine the sensitivity of function exponents and regression coefficients of variation (R^2) to the procedure employed. The latter test is motivated by the need to repeat Wheeler's (1984) demonstration of the importance of coordinate system placement, which is inconclusive owing to the use of Doornkamp and King's (1971) flawed equation (Harbor and Wheeler, 1992).

The polynominial model applied by this study does not require designation of valley-bottom points, because the coordinate system is defined by a point high on the valley side. Nevertheless, this model was applied to the upper Bear Valley sections with and without valley-bottom data to test model sensitivity, and to two other sections to isolate distinct morphological components. On most transects, the upper limit of valley sides was set at ice margin elevations based on field evidence of lateral moraines or upper erratic limits on one side, assuming an equal ice level on the opposite side. The glacial advance used for reference was not the most extensive in these valleys but represents the latest glaciation of these valleys and is presumed to be chronostratigraphically correlated with the last glacial maximum. Upper limits of the first and third Yuba Gorge transects were set at distinct topographic breaks in side-slope below and above the Tioga ice surface, respectively. Upper limits in Tenaya Canyon are based on ice margins of the 'Wisconsin' stage shown on Matthes' (1930) map.

CURVE-FITTING METHODS

Linear regressions were performed on transformations of the station-elevation data to derive mathematical

functions of valley morphology. All functions were tested for computation errors by summing regression residuals to zero (Draper and Smith, 1966). For power functions, data manipulation began with identification of the coordinate system. Centre points in bedrock-floored valleys were located at the valley thalweg. In the two Bear Valley alluvial sections, centre points were located midway between the two valley sides defined at the ice surface level, and midpoint elevations were defined at the minimum elevation. Data were reordered by setting lateral distances to zero at valley centres and increasing outward. Elevations were referenced to the lowest point on each half transect by subtracting the minimum elevation. Finally, zero elevation and station data points were eliminated to prevent log-of-zero errors, and logarithms were taken of both station and elevation data.

Power functions were derived for each half valley by univariate linear regression on the log-transformed values (Svensson, 1959; Draper and Smith, 1966; p. 132; Doornkamp and King, 1971; King 1974; Harbor and Wheeler, 1992):

$$\log H = g + f \log X \qquad (3)$$

Exponentiating both sides of Equation 3 yields the power function of Equation 1. Heights predicted by Equation 1 can be converted to elevations by adding the elevation of the minimum point.

Polynomial functions were derived by regression of elevation above sea level on both station and station squared (independent variables in Equation 2), where station is lateral distance from the left-side datum. It is not necessary to split samples, zero and adjust elevation data, identify valley minima, or specify valley-bottom coordinates, but only to identify beginning and ending station points, reference the station data to zero at the first station, and calculate squares of the station data. With the enhanced availability and capability of computers to perform the analysis, this technique requires less effort than power functions because it obviates most data pre-processing.

POWER FUNCTION RESULTS

Most regressions for both models were significant ($\alpha = 0{\cdot}001$), and explained variance (R^2) was typically greater than 90 per cent (Tables II and III). Elevation data along transects have strong serial correlations (Doornkamp and King, 1971; p. 284), however, so significant regressions are common, even when systematic residual distributions indicate an inappropriate model. Explained variance is reported to provide a relative measure of the success of models at curve-fitting rather than to test hypotheses at a given significance level.

All contour data

Power functions derived using all contour data yielded statistically significant regressions, with all but one of the 14 functions having explained variances greater than 90 per cent (Table II). Three categories of valley functions can be identified: (1) the four broad upper Bear Valley profiles with high exponents (1·6 to 3·3); (2) the stepped profiles of Tenaya Canyon, lower Bear Valley, SY1-R, and SY2-L with low exponents (0·8 to 1·05); and (3) the other South Yuba profiles which are steep with moderate exponents (1·15 to 1·6). However, many functions have exponents that do not represent the overall valley morphology, owing to domination by a few valley-bottom data points. For example, the four non-stepped South Yuba profiles have overall morphologies very close to linear; the SY1-L and SY2-R overall profiles are slightly concave-upward, while both SY3 overall profiles are slightly convex-upward (Figures 4 and 5). Yet only the exponent for SY2-R is of the appropriate sign and magnitude (1·15) for the observed form. The two convex-upward SY3 profiles intuitively should have exponents less than one (Figure 5A), but sample points near the channel dominate the power function regressions and both models express a concave-upward form. The log–log plot of the left profile illustrates how the regression is dominated by subtle topographic changes within 30 m of the valley bottom (Figure 5B). This corroborates Harbor and Wheeler's (1992) conclusion that power functions are very sensitive to local form components of valley bottoms. It also suggests a lack of reliability of power function exponents as indicators of overall valley form, in spite of high coefficients of determination.

Power functions for the two alluviated Bear Valley transects yield high explained variances, and appear to be good functional expressions of these sections. Exponents of functions for the right side are representative

Table II. Power functions

ID	Side	Method	Function	R^2	N
BV1	L	All,Ctr2000	$0{\cdot}0000101X^{2{\cdot}44}$	0·950	24
BV1	R	All,Crt2000	$0{\cdot}000670X^{1{\cdot}82}$	0·977	25
BV1	L	All,Crt1842	$0{\cdot}00143X^{1{\cdot}77}$	0·972	24
BV1	R	All,Crt1842	$0{\cdot}00971X^{1{\cdot}46}$	0·973	25
BV1	L	All,Crt2000,Or@50	$0{\cdot}0330X^{1{\cdot}29}$	0·979	24
BV1	R	All,Crt2000,Or@50	$0{\cdot}0983X^{1{\cdot}13}$	0·948	25
BV1	L	All,Crt1842,Or@50	$0{\cdot}514X^{0{\cdot}917}$	0·961	24
BV1	R	All,Crt1842,Or@50	$0{\cdot}541X^{0{\cdot}901}$	0·931	25
BV1	L	Interp'd Pts	$0{\cdot}00306X^{1{\cdot}56}$	0·823	37
BV1	R	Interp'd Pts	$0{\cdot}0182X^{1{\cdot}33}$	0·935	30
BV1	L	NoC1475	$0{\cdot}0000157X^{2{\cdot}37}$	0·989	17
BV1	R	NoC1475	$0{\cdot}0000108X^{2{\cdot}42}$	0·99967	19
BV1	L	NoC1475,Or@50	$0{\cdot}000502X^{1{\cdot}89}$	0·981	17
BV1	R	NoC1475,Or@50	$0{\cdot}00111X^{1{\cdot}79}$	0·998	19
BV2	L	All	$0{\cdot}0000000138X^{3{\cdot}30}$	0·989	25
BV2	R	All	$0{\cdot}00489X^{1{\cdot}61}$	0·996	26
BV2	L	Interp'd Pts	$0{\cdot}0183X^{1{\cdot}29}$	0·804	36
BV2	R	Interp'd Pts	$0{\cdot}00359X^{1{\cdot}60}$	0·956	30
BV2	L	NoC1450	$0{\cdot}000000343X^{2{\cdot}84}$	0·991	19
BV2	R	NoC1450	$0{\cdot}0112X^{1{\cdot}48}$	0·996	21
BV2	L	NoC1450,Or@50	$0{\cdot}0000536X^{2{\cdot}16}$	0·995	19
BV2	R	NoC1450,Or@50	$0{\cdot}139X^{1{\cdot}38}$	0·993	21
BV3*	L	All	$0{\cdot}467X^{0{\cdot}889}$	0·962	26
BV3*	R	All	$2{\cdot}041X^{0{\cdot}731}$	0·910	34
BV3*	L	NoC1380	$0{\cdot}0810X^{1{\cdot}14}$	0·876	17
BV3*	R	NoC1380	$14{\cdot}8X^{0{\cdot}418}$	0·931	20
SY1	L	All	$0{\cdot}0272X^{1{\cdot}60}$	0·953	29
SY1*	R	All	$0{\cdot}198X^{1{\cdot}05}$	0·877	23
SY1	L	NoC1330	$0{\cdot}108X^{1{\cdot}35}$	0·989	22
SY1*	R	NoC1330	$0{\cdot}000144X^{2{\cdot}17}$	0·984	16
SY2*	L	All	$0{\cdot}590X^{0{\cdot}994}$	0·908	42
SY2	R	All	$0{\cdot}251X^{1{\cdot}15}$	0·972	38
SY2*	L	NoC1540	$33{\cdot}52X^{0{\cdot}367}$	0·929	13
SY2	R	NoC1540	$0{\cdot}286X^{1{\cdot}14}$	0·993	9
SY3	L	All	$0{\cdot}218X^{1{\cdot}17}$	0·946	43
SY3	R	All	$0{\cdot}0428X^{1{\cdot}46}$	0·958	42
SY3	L	NoC1250	$0{\cdot}924X^{0{\cdot}936}$	0·998	25
SY3	R	NoC1250	$0{\cdot}528X^{1{\cdot}05}$	0·99913	24
Ten*	L	All	$2{\cdot}64X^{0{\cdot}804}$	0·943	38
Ten*	R	All	$2{\cdot}93X^{0{\cdot}791}$	0·982	37
Ten	L	Interp,Or@Btm	$86{\cdot}0X^{0{\cdot}229}$	0·570	38
Ten	R	Interp,Or@Btm	$95{\cdot}5X^{0{\cdot}210}$	0·559	37
Ten	L	NoC1870	$0{\cdot}772X^{0{\cdot}999}$	0·964	22
Ten	R	NoC1870	$0{\cdot}753X^{0{\cdot}999}$	0·922	22
Ten	L	NoC1870,Or@Bench	$0{\cdot}000192X^{2{\cdot}21}$	0·994	25
Ten	R	NoC1870,Or@Bench	$0{\cdot}000129X^{2{\cdot}27}$	0·969	25

* Stepped profiles
All: all contour data used for sample
Interp: valley-bottom data interpolated assuming a flat bottom
NoC*xxx*: no centre points used in curve fitting; data omitted below *xx* m elevation
NoC*xxx*,Or@50: no centre data points used; vertical axis origin lowered 50 m
Or@Btm,Or@Bench: vertical axis origin set at channel bottom or bench
@2000 or @1842: horizontal centre point used in BV1
R,L: right and left side (looking downvalley), respectively

Table III. Polynomial functions

ID	Method	Elevation functions ($a + bS + cS^2$)	R^2	N
BV1	All	$1653 - 0{\cdot}476S + 0{\cdot}000208S^2$	0·988	52
BV1	NoC1475	$1663 - 0{\cdot}547S + 0{\cdot}000238S^2$	0·994	36
BV2	All	$1667 - 0{\cdot}524S + 0{\cdot}000222S^2$	0·996	53
BV2	NoC1500	$1667 - 0{\cdot}511S + 0{\cdot}000217S^2$	0·996	30
BV3	All	$1609 - 0{\cdot}364S + 0{\cdot}000161S^2$	0·812	61
BV3	NoC1380	$1598 - 0{\cdot}291S + 0{\cdot}000123S^2$	0·846	50
BV3	NoC1430–1500*	$1592 - 0{\cdot}236S + 0{\cdot}000106S^2$	0·787	37
SY1	All	$1621 - 1{\cdot}149S + 0{\cdot}000896S^2$	0·932	53
SY1	NoC1330	$1617 - 1{\cdot}070S + 0{\cdot}000827S^2$	0·926	38
SY2	All	$1813 - 0{\cdot}790S + 0{\cdot}000328S^2$	0·623	81
SY2	NoC1560	$1690 - 0{\cdot}281S + 0{\cdot}000120S^2$	0·823	21
SY2	C < 1540†	$1577 - 0{\cdot}386S + 0{\cdot}001427S^2$	0·961	59
SY3	All	$1639 - 1{\cdot}289S + 0{\cdot}000803S^2$	0·916	86
SY3	NoC1250	$1579 - 0{\cdot}923S + 0{\cdot}000578S^2$	0·973	49
Ten	All	$2222 - 1{\cdot}357S + 0{\cdot}000940S^2$	0·939	77
Ten	NoC1870	$2194 - 1{\cdot}118S + 0{\cdot}000775S^2$	0·980	44

* 1430–1500 are elevation limits (m) of left and right sides, respectively
† $C < 1540$ is the inner gorge only (below 1540 m) of SY2

of values commonly reported for intensively glaciated valleys (1·6 and 1·8), but the left side of Bear Valley has abnormally large exponents (2·44 and 3·30 for BV1 and BV2, respectively). Mass wasting deposits at the base of slopes, which are very subtle on aerial photographs, strongly influence functions on BV1 (Figure 6A). A representative log–log plot of the right side of the valley (Figure 6B) shows that the model fits the log data

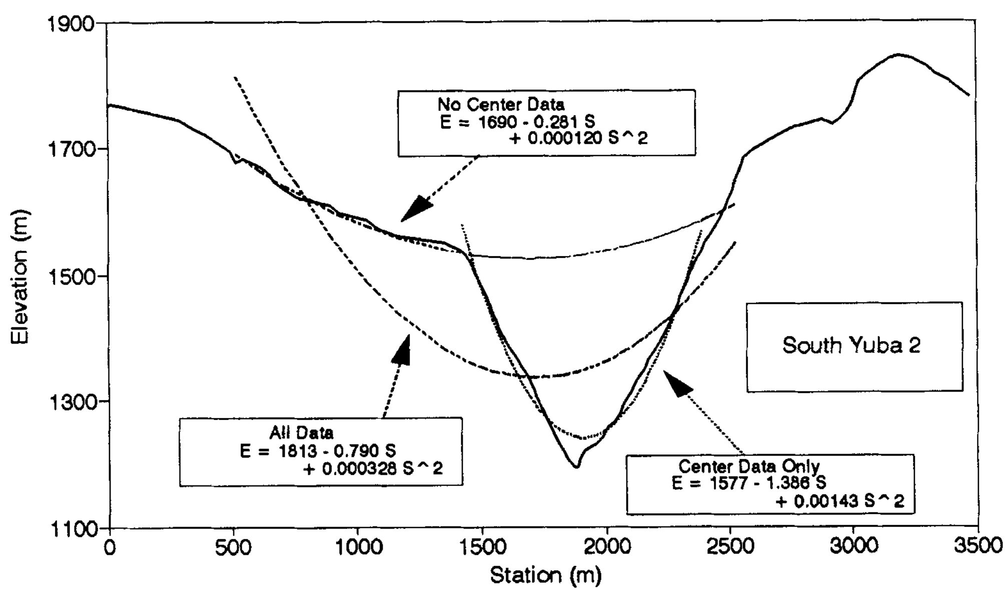
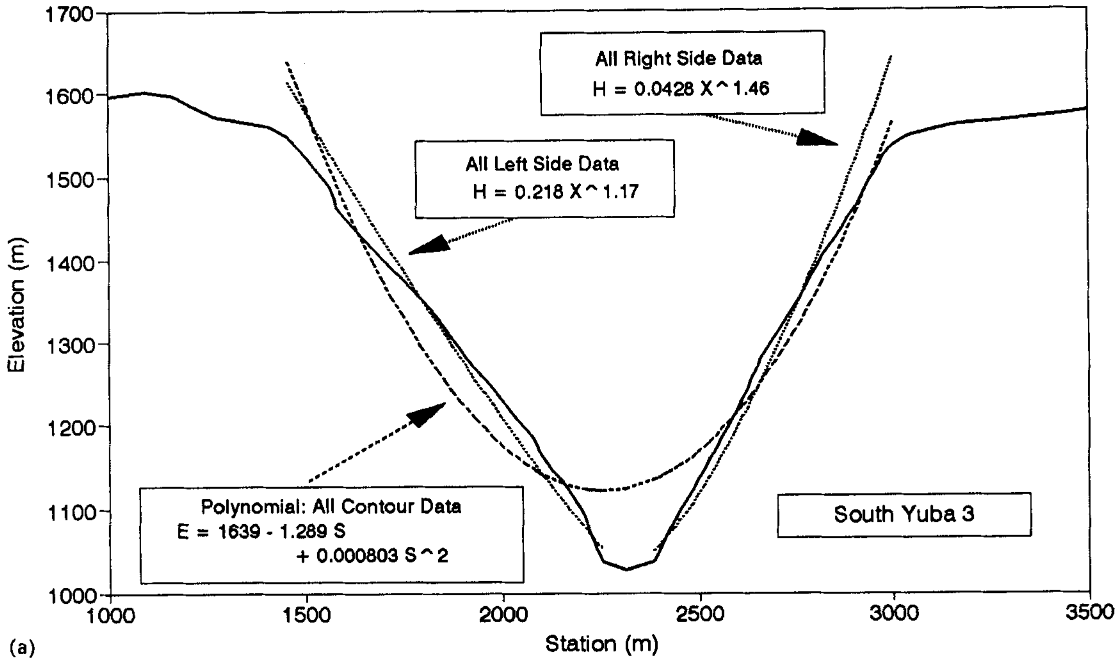


Figure 4. Yuba Gorge Transect 2 with polynomials derived using all data, no data below 1560 m, and no data above 1525 m. The composite valley polynomial is not an acceptable model, but the other two demonstrate valley partitioning into two component parts



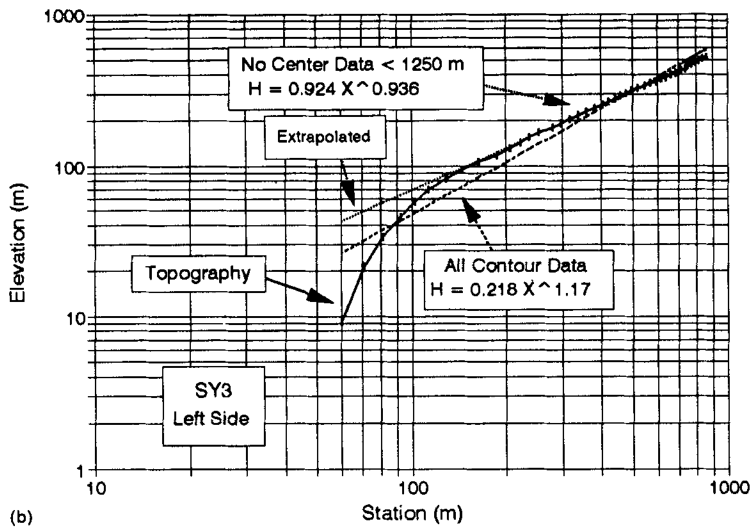


Figure 5. Yuba Gorge Transect 3, an over-deepened canyon with slightly convex-upward side-slopes. (A) Power functions using all contour data have exponents > 1·0, indicating concave-upward slopes due to microtopography at slope base. Owing to convex side-slopes, the polynomial gives a poor approximation of valley form at this site. (B) Log–log plot of left side power function reveals the strong influence of valley-bottom topography on the power function

beyond 100 m fairly well, but the regression line is pulled up from the centre by a few points on the colluvial surface. This valley-bottom influence lowers the regression line slope and underestimates elevations at the valley rim in a manner similar to the example presented by Harbor and Wheeler (1992, Fig. 2). Many data points up on the valley wall are compressed on the log plot into the small segment at stations greater

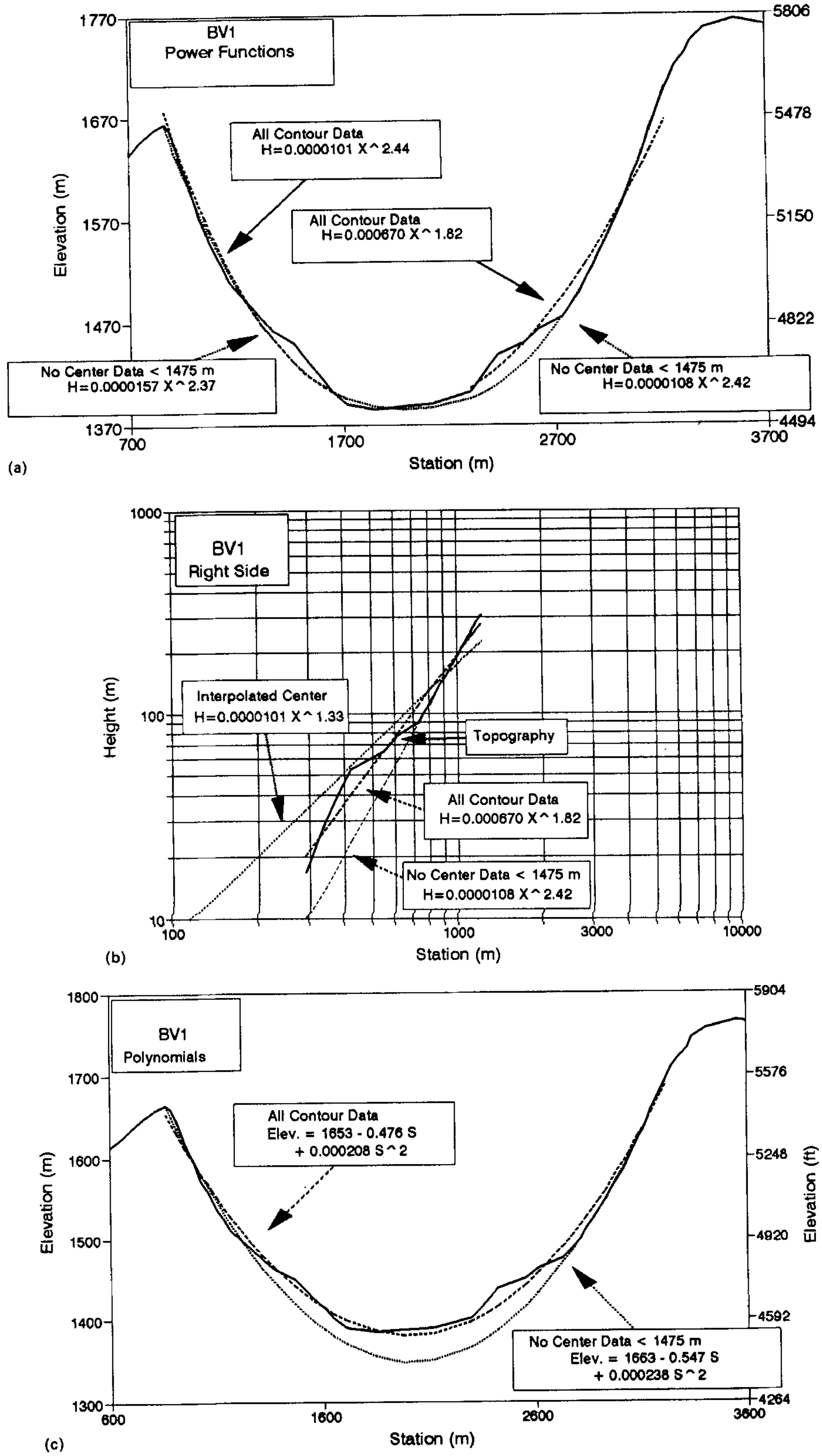


Figure 6. Bear Valley Transect 1. (A) Power function derived using all contour data, and no data below 1475 m. (B) Log–log plot of right side power function showing strong influence of valley-bottom topography. (C) Polynomials derived using all data and no data below 1475 m

than 900 m and, because their log-errors are small, they fail to pull the regression line up to higher elevations. Both functions at BV2 have less colluvium, smaller regression errors and high explained variance, and provide good models of valley morphology at that site, is spite of the alluvial fill.

Power functions for the bedrock-floored Tenaya Canyon are convex-upward ($f \leq 0{\cdot}804$) owing to the inner gorge, which is eroded about 200 m into the base of a broader trough (Figure 7A). Convex-upward power functions were also derived for the other stepped profiles (BV3 and SY2-L) except for SY1-R, which came out essentially linear ($f = 1{\cdot}05$). While these models of stepped profiles are flawed by systematic errors, there is no simple alternative model and stepped profiles should be avoided for morphological analyses of this nature.

Centre data augmented by interpolation

Power function regressions were not improved by a more complete sample representation of alluvial valley bottoms. In fact, the addition of interpolated points degraded power function results considerably for the two upper Bear Valley transects in terms of both explained variance and systematic error (Figure 6B). Exponents decreased in all cases except for BV2-R, which remained constant. This empirically verifies the relationship shown schematically by Harbor and Wheeler (1992) in which an alluvial valley floor distorts the function as a result of biased control by points close to the origin. The loss of data representation due to sample bias of the contour method was beneficial in the previous case, because data points on the valley flat do not conform to the power function model, yet are highly influential to its derivation. Greater representation of valley bottoms does not resolve but exacerbates difficulties in fitting the power functions.

Data were also augmented in an attempt to simulate a hypothetical flat floor in Tenaya Canyon. When used with the original valley bottom as the elevation axis origin, interpolated points failed to produce viable power functions (Figure 7B). The curves are forced toward the valley bottom because height must equal zero at station zero (Equation 1). Yet non-zero points near the origin pull up the lower end of the regressing line in a manner similar to that illustrated in Figure 6B, resulting in greatly reduced exponents (0·23 and 0·21). This provides a graphic example of the control exerted on functions by points near the origin and assignment of the coordinate system.

Centre data omitted

Power functions derived using no valley-bottom data increased explained variance in most cases and improved models in some cases, but some of the resulting functions remain inappropriate models. For example, Tenaya Canyon functions, which were convex-upward with all contour data, became linear ($f = 0{\cdot}999$) when centre data were omitted below an elevation of 1870 m, but the new models under-predict elevations for much of the valley bottom (Figure 7A).

Power functions were successfully derived by omitting points on the two upper Bear Valley sections. A pair of power functions derived for BV1 using only data above the colluvium at 1475 m produced profiles with high explained variances (Figure 6A; Table II). The left-side exponent changed very little through the omission of points, but the right-side exponent is much larger. The exponents (2·49 and 2·33) on both sides are higher than those commonly reported. The power function on the left side of BV2 also yielded increased explained variance by eliminating valley-centre data, but extrapolation of the regression line predicts valley-bottom elevations 5 to 15 m higher in places than the present surface. Although coefficients of determination increased, smaller sample sizes and ranges offset the benefits. Furthermore, predicted valley-bottom elevations remain dependent on the specification of the coordinate system.

Alternative coordinate systems

To test the sensitivity of power function exponents to coordinate system locations as part of an evaluation of the Aniya and Welch (1981) iterative method, three sections were subjected to recalculation of power functions using vertically or horizontally shifted coordinates. Power functions were recalculated with origins of the coordinate systems arbitrarily reset to 50 m below a raised surface using (1) all contour data and (2) no

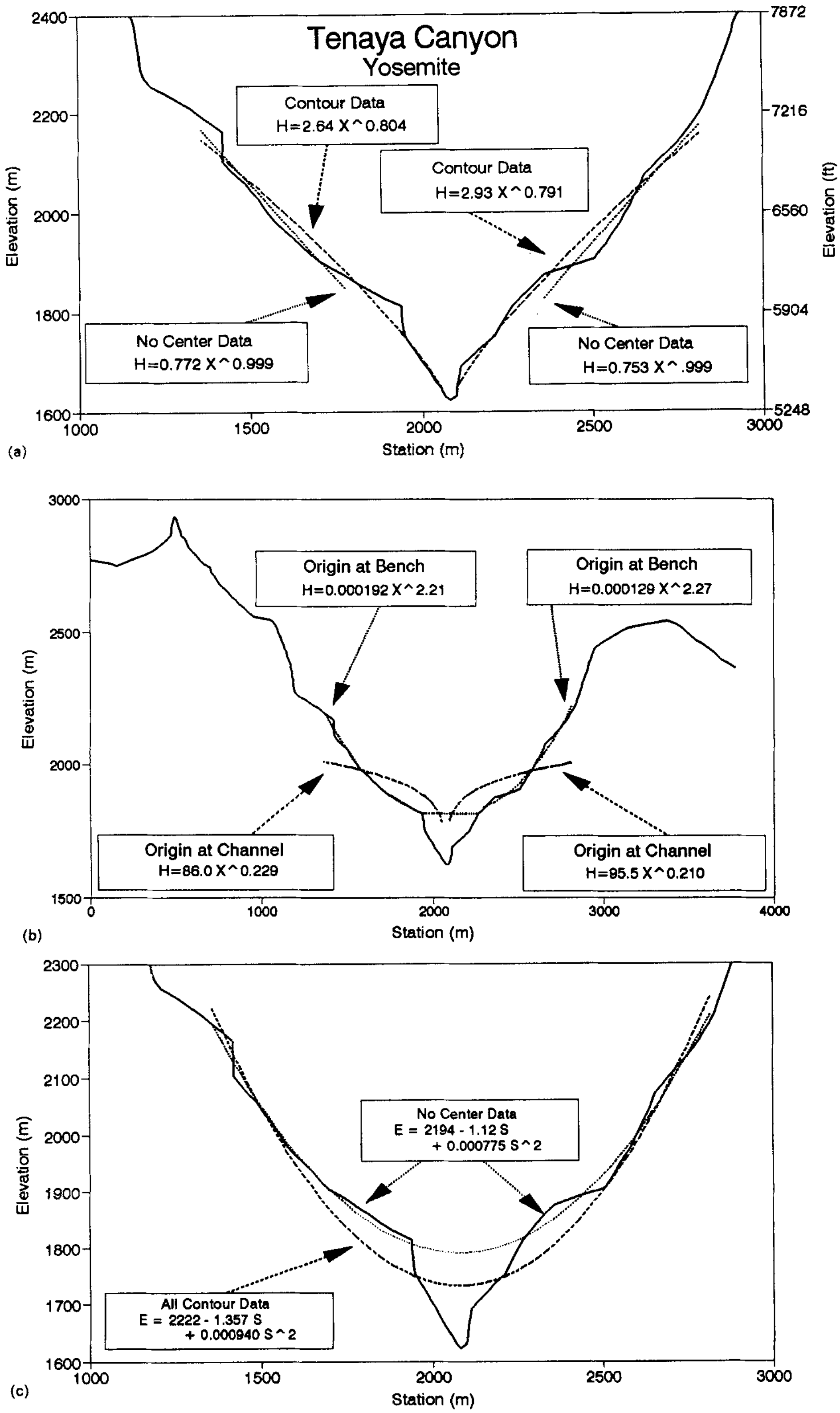


Figure 7. Transect across Tenaya Canyon. (A) Power functions derived using all contour data and no data below 1870 m. (B) Power functions derived using data interpolated across the upper bench surface. Placement of the coordinate origin is critical. (C) Polynomials for all data, and no data below 1870 m, illustrating partitioning

centre data below 1475 m at BV1, or below 1450 m at BV2 (Table II). Elevations were incremented by 50 m and logged, and regressions were run as before.

Redefinition of the vertical axis resulted in substantial decreases in power function exponents and increases in coefficient d in all eight trials (Table II). Explained variance for functions generally decreased slightly with the lowered origin, although it increased in a few functions. Origins were also shifted horizontally at BV1 by 158 m, from the valley centre at 2000 m to a centre at 1842 m corresponding to the present valley thalweg identified by a non-linear visual contour interpolation. Resulting power functions have substantially different exponents, while changes in explained variance are relatively small (Table II). Placement of the coordinate system is a dominating factor determining power function exponents, even when no valley-bottom data are included in the analysis. Yet subtle changes in regression parameters cast doubt on the potential for identifying the appropriate function by iterations to maximize strength of regressions, as advocated by Aniya and Welch (1981).

To partition slopes using the power function model, it is necessary to identify the elevation of the hypothetical surface in advance by referencing the vertical axis origin to zero at the surface. In Tenaya Canyon, power functions were derived by redefining the vertical origin at the top of the bench surface. The resultant functions describe the upper valley very well, with explained variance of 97 and 99 per cent and exponents of 2·2 (Figure 7B; Table II). The requirement to specify the valley-bottom elevation, however, limits the use of the power model as an objective method for identifying morphological components of complex valleys. A preferred function would be derived by a curve-fitting technique that utilizes valley-side data independently of the position of the valley bottom.

Discussion

Power functions fitted to 14 half-valley forms by several different techniques produced exponents ranging from 0·21 to 3·30. High exponents of wide Bear Valley profiles and low exponents for the narrow non-stepped South Yuba profiles corroborate the validity of using power function exponents as a general measure of valley morphology, if used with caution and standardized procedures. However, several fundamental limitations to the use of power functions as indicators of overall valley shape exist. First, the log-least-squares procedure is strongly influenced by local valley-bottom morphology, which can suppress the influence of overall valley form (Harbor and Wheeler, 1992). Second, the need to locate the coordinate system precisely at the valley-bottom centre poses problems in many valleys. Sensitivity of Bear Valley power function exponents to shifts in the coordinate system origin indicates that exponent values are not a robust measure of overall valley form. Third, power functions are sensitive to the sampling method employed; quite different results were obtained by fitting functions to different valley-bottom data permutations. These three sources of variation indicate that exponent values are subject to arbitrary operational decisions and sampling artifacts that can preclude comparisons between studies. Even where the bedrock surface is exposed and easily identified, exponents may vary in response to factors independent of glacial processes.

The inference of glacial erosion processes from valley morphology is complicated by non-morphological factors such as lithology (Augustinus, 1992a). These difficulties are compounded by form uncertainties when interpretations are based on power function exponents. Exponents are highly sensitive to valley-bottom morphology which is prone to alterations of post-glacial fluvial, colluvial and other non-glaciogenic processes. Exponents are also sensitive to coordinate placement, and there is no theoretical rationale for selecting the iterative or any other method of deriving functions.

It would be inappropriate to interpret the high Bear Valley exponents as the result of more intensive glaciation of Bear Valley. Both valleys were subjected to at least two major glacial advances, but the shallow, gently sloping Bear Valley is an exhumed palaeovalley formerly filled with unconsolidated lahars, while the steep Yuba Gorge is developed in granite and has been deepening rapidly (James, in press). Furthermore, Bear Valley narrows and steepens at and below BV3 corresponding to changes in bedrock and slope. Valley gradients and lithology appear to be very important in explaining the dichotomy in valley shapes between Bear Valley and Yuba Gorge rather than duration or other measures of glacial intensity. If fact, Yuba Gorge was more intensely glaciated than Bear Valley in terms of depth, ice velocities and discharges, yet it has much lower power function exponents.

Contour sample under-representation of valley bottoms did not pose a problem for power functions in sediment-filled sections, where the addition of points seriously degraded regressions. In fact, *omission* of valley-bottom data increased explained variance in most cases. Owing to the sensitivity of power functions to valley-bottom data, better results for overall valley morphology can be obtained by minimizing this information, so that the valley-wall morphology can exert the dominant control. Valley-bottom data that are included should be precisely measured, but these points can exert such a dominant control on functions that the resulting model fits poorly up on the valley sides. Thus, valley-bottom data are best used where the objective is to model the basal ice configuration rather than to characterize overall valley morphology.

Stepped valley sections cannot be modelled by any single simple function, so it is not surprising that power functions were unsuccessful in modelling these forms. For deeply notched valley floors, functions based on all contour data are generally linear or convex-upward ($f \leq 1{\cdot}05$). This remained true with the omission of valley-bottom data except at SY1-R, where points on a broad rock bar were omitted. Aniya and Welch (1981) applied power functions to stepped profiles by dividing side slopes into two segments and applying separate functions to each segment. This technique was not attempted because using four functions to express overall morphology of a single valley transect is unwieldy and does not avoid the problems previously outlined.

In short, power functions can fit a wide variety of forms, but they are very sensitive to valley-bottom features, they require precise specification of the coordinate system, which is often unknown, they are incapable of objectively partitioning complex valleys into separate elements, and sample sizes are greatly reduced by dividing valleys into halves. It is desirable, therefore, to have a model that is robust to subtle valley-bottom changes, is not based on a coordinate system centred on the valley bottom, expresses total valley form, utilizes the entire sample, and can be used to partition polymorphic valley components.

POLYNOMIAL MODEL RESULTS

The goodness of fit of power and polynomial functions should not be compared simply by evaluation of regression statistics, because polynomials include both valley sides in one function of overall form. They provide both parameters required to evaluate overall valley form: valley wall curvature and width.

All contour data

Polynomial functions derived for the two upper Bear Valley transects using all contour data have explained variance as high as or higher than variance explained by power functions using the same data (Table II and III). In addition, sample sizes are doubled, and regression errors are reasonably well distributed. For example, the first Bear Valley transect (BV1) yields an explained variance of 98·8 per cent with a distribution of regression errors that is only systematic at sites of post-glacial erosion and deposition (Figure 6C). Results for BV2 are similar. This good polynomial fit to a valley eroded into incompetent rock free of structural controls suggests that the function may in some cases approximate a form adjusted to glacial processes.

In bedrock valleys, the success of polynomial functions must be considered in the context of various degrees of convexity, asymmetry or polymorphism. The lower Bear Valley section (BV3) is stepped and asymmetric, and the polynomial model is inappropriate. In Tenaya Canyon, the stepped surface could not be accurately modelled by the polynomial function (Figure 7B), but valley symmetry and bench accordance allow it to be partitioned as is demonstrated in the next section.

Fitting polynomials to transects in the Yuba Gorge met with variable success since non-parabolic sections can only be approximated by the quadratic function. On transect SY1, asymmetry due to a broad, flat, rock bar resulted in poor fit and the function provides only a first approximation of valley geometry ($R^2 = 93$ per cent). Transect SY2, a deep, narrow, inner gorge cut into one side of a broad, shallow higher valley (Figure 4), cannot be modelled by a single simple function, but is partitioned using polynomials in the next section. Convex-upward valley-side slopes on transect SY3 result in systematic errors which limit the polynomial

function validity to a first approximation of valley form (Figure 5A). In this case, the pair of power functions provided superior models, although their exponents imply upward concavity.

Centre data omitted and valley partitioning

Omission of valley-bottom data improved many polynomial models both by increasing explained variance (Table III) and by reducing systematic errors. This technique allows valley walls to provide the dominant signal and ignores valley-bottom forms completely. On BV1 the function fits valley walls very well, and explained variance increased from 98·8 to 99·4 per cent (Figure 6C); however, the predicted 39 m depth of valley alluvium appears excessive given bedrock outcrops exposed elsewhere in the outwash. This may be due to post-glacial effects. Not only has the valley bottom been alluviated, but also upper valley walls of unconsolidated andesitic breccia have been eroded. On BV2 the response to data limitation was more subtle, with little change in regression strength or model coefficients. The predicted maximum depth to bedrock on this transect is about 4 m below the present alluvial floor. Polynomial functions were derived for BV3 in two permutations, omitting valley-bottom data at two levels. As a result of asymmetry, stepped surfaces and discordant benches, these functions are not viable and partitioning is not feasible.

The centre-omission method allows the use of polynomials to partition polymorphic forms. Omission of valley-bottom points below 1870 m in Tenaya Canyon produces a viable polynomial model that not only explains 98 per cent of the variance in elevation, but also describes a surface corresponding to the bedrock bench distinct from the lower inner canyon (Figure 7C). This method provides an objective means of separating the two canyon forms, regardless of the genetic interpretation. Polynomials were also used to differentiate between upper and lower surfaces in SY2. In contrast to the overall polynomial model for this section, separate models for an upper and lower portion of the canyon, split at 1525 m elevation, are statistically strong and viable (Figure 4). This partitioning method allows the application of equations presented in a later section to calculate geometric characteristics of the valley components.

Discussion

Polynomial functions are valuable as a succinct and explicit model of the entire valley form. As with power functions, polynomials can provide valid and statistically significant expressions of glacial valley form. These functions not only fit the data well, but they also provide a substantial amount of morphological information that is explored in the next section. Quadratic equations express a parabolic form and are inappropriate for convex-upward side slopes, asymmetrical valleys and stepped slope profiles. Although higher-order polynomials can express more complex forms, principles of parsimony and model sensitivity may preclude their use. Further, it may be difficult to interpret the many terms in high order expressions (Augustinus, 1992a). Alternative functions that treat valley sides individually (e.g. power functions) may be superior models for convex-upward and asymmetrical valleys.

Where asymmetry is the result of two form components, such as a notch cut into one side of a broad, high valley, it may be possible to partition the valley objectively into its component parts using polynomials. Isolation of upper and lower valley components in this manner may prove useful not only for geometric measurements and model parameterizations, but also for the identification of polycyclic or multistadial surfaces.

VALLEY QUADRATIC PARAMETERS

Quadratic equations have several useful properties, and if a second-order polynomial can be fitted to a valley cross-section in a statistically unbiased and significant manner, these properties can be used to describe the valley geometry. For example, coefficient a in equation 2 is the elevation of the datum used to describe the coordinate axis, coefficient b is slope at that point, and coefficient c provides shape information for the curve. Polynomial coefficients for the seven sections are related to overall valley form (Table III). The broad, shallow BV1 and BV2 sections have low absolute values of b and c while the deep, narrow SY3 and SY2 inner Gorge and Tenaya Canyon sections have high absolute values.

Table IV. Valley parameters from polynomial functions

ID	Data Elev. range (ft)	E_{min} $a - b^2/4c$ (m)	Top width $-b/c$ (m)	Area $-b^3/6c^2$ (m^2)	Mean depth $b^2/6c$ (m)	Max. depth $b^2/4c$ (m)	Form ratio $-b/4$ (m)	Boundary length (m)	Radius: area/length
BV1	All	1381	2288	415481	182	272	0·119	2372	175
BV1	$\geq$ 1475	1349	2298	481577	210	314	0·137	2408	200
BV2	All	1358	2360	186570	207	309	0·131	2464	197
BV2	$\geq$ 1500	1366	2355	472281	201	301	0·128	2454	192
SY1	All	1253	1282	314921	246	368	0·287	1525	206
SY2	$\geq$ 1540	1525	2342	256811	110	165	0·070	2372	108
SY2	$<$ 1540	1240	971	217921	225	337	0·347	1226	178
SY3	All	1122	1605	553586	346	517	0·322	1977	280
Ten	$\geq$ 1870	1791	1443	387775	269	403	0·280	1703	228

Systematic changes in polynomial coefficients can be exploited by deriving geomorphic parameters from the quadratic equations. Several morphometric parameters can be mathematically determined from the coefficients of the quadratic equation. These parameters can be used for geometric or volumetric calculations of trough dimensions, for inputs to glaciological or geomorphic models, and for testing process-response hypotheses. Representative values of the following parameters from the study transects are presented in Table IV.

Some parameters can be analytically derived from the first derivative of the elevation function defined in Equation 2:

$$dE/dS = b + 2cS \tag{4}$$

where dE/dS is valley-side slope at any point at a horizontal distance S from the datum. At the datum S is zero and b equals slope. At the valley centre elevation reaches a minimum and slope is zero so:

$$S_m = -b/2c \tag{5}$$

where S_m is distance to the valley midpoint (parabolic vertex). The midpoint of the quadratic is the lowest point, so substitution of Equation 5 into Equation 2 yields an expression for the minimum elevation on the transect (E_{min}):

$$E_{min} = a - b^2/4c \tag{6}$$

Elevation of the vertical datum (coefficient a) is on a linear scale, so changes to it will shift the function up or down without altering the shape or size of the model cross-section. For example, subtraction of the minimum elevation references the transect to an elevation of zero at the valley bottom.

With the coordinate system on the valley wall, a number of depth and shape parameters can be derived. Doubling the midpoint value of Equation 5 yields the valley top width (W):

$$W = -b/c \tag{7}$$

Valley depth can be calculated at any horizontal location by defining the valley top as a flat surface level with the initial point. Maximum elevation is given by coefficient a, so depth (D) at any point S is found by subtracting elevation at the point from the first coefficient:

$$D = -bS - cS^2 \tag{8}$$

In a similar manner, maximum depth (D_{max}) can be calculated using Equation 6 to subtract minimum elevation from the first coefficient:

$$D_{max} = b^2/4c \tag{9}$$

The polynomial equivalent of form ratio (F) is the ratio of maximum depth to width:

$$F = -b/4 \tag{10}$$

An expression for valley cross-section area (A) can be derived by integrating valley depth (Equation 8) in respect to distance across the width of the valley:

$$A = \int_0^w (-bS - cS^2)\mathrm{d}S = -[bS^2/2 + cS^3/3] \,|_0^{-b/c} = -b^3/6c^2 \tag{11}$$

Mean depth (D_{Mn}) is the ratio of cross-section area to top width:

$$D_{Mn} = (-b^3/6c^2)/(-b/c) = b^2/6c \tag{12}$$

A comparison of Equations 12 and 9 reveals that mean depth is equal to 2/3 maximum depth in quadratic valleys.

The boundary length of the ice–land contact (L), useful for modelling erosion and boundary shear stress, can be calculated by integrating the hypotenuse of the differentials of elevation and lateral distance with respect to distance across the transect:

$$L = \int_0^w [1 + (\mathrm{d}E/\mathrm{d}S)^2]^{1/2}\mathrm{d}S = \int_0^w [1 + (b + 2cS)^2]^{1/2}\mathrm{d}S \tag{13}$$

A quality of the second-order polynomial is that this integral can be solved using the inverse hyperbolic function $\sinh^{-1}$ and simplified to an expression of boundary length in terms of coefficients b and c:

$$L = 1/2[ln((1 + b^2)^{1/2} - b) - \mathrm{b}(1 + b^2)^{1/2}] \tag{14}$$

Valley radius, which is analogous to hydraulic radius and can be used to compute average boundary shear stress, can be calculated as the ratio of valley cross-section area to boundary length using the results of Equations 11 and 14. A comparison of mean depth with valley radius values on Table IV reveals that radius approaches mean depth as valleys become wider and shallow (BV1 and BV2), but is considerably smaller than mean depth for the deep and narrow valleys.

The shape factor (f), equal to the product of the trough-section area and boundary perimeter length divided by the maximum depth (Augustinus, 1992), can be derived utilizing Equations 9, 11 and 14:

$$f = AL/D_{max} = -b/3c^2[ln((1 + b^2)^{1/2} - b) - b(1 + b^2)^{1/2}] \tag{15}$$

CONCLUSIONS

Power and polynomial functions were tested on seven glaciated Sierra Nevada valleys using three cross-section sample methods: (1) all contour data; (2) contour data augmented by interpolation across valley bottoms; and (3) all valley-bottom data omitted. As expected from previous studies, power functions yielded concave-upward forms for most non-stepped profiles. However, function exponents were unreliable indicators of overall valley form. Function exponents for non-stepped profiles ranged widely from 1·15 to 3·3, but in many cases were mathematical artifacts of valley-bottom microtopography, selection of a coordinate system, or sample methods, rather than overall valley form. Sensitivity of power function exponents to local valley-bottom morphology corroborates the findings of Harbor and Wheeler (1992), limits their reliability as expressions of overall valley form, and suggests that the greatest utility of power functions lies in describing conditions in the basal segment of the profile.

Attempts to develop more appropriate functions of overall valley form in alluviated troughs by increasing

sample sizes on valley bottoms failed. In fact, the best power function models of the two alluviated valleys were obtained by omitting all valley-bottom and toe-slope data from the curve-fitting process. Power functions remained dependent on the precise specification of the coordinate system origins, however, which can be difficult to validate with data drawn from contour maps.

As with all methods of inferring glacial erosion processes from valley morphology, non-glaciogenic factors limit the validity of inferring intensity of glaciation from power function exponents. The two upper Bear Valley sections are much wider and shallower that the others and have much higher power function exponents than sections in Yuba Gorge. However, both valleys appear to have had similar glaciation chronologies, and morphological differences are presumably more a function of lithology and valley gradient than glacial history in accordance with the findings of Augustinus (1992a). Such non-glaciogenic factors that influence valley morphology also complicate the inference of process from polynomial parameters. With polynomial functions, however, there may be less mathematical uncertainty about valley shape.

Second-order polynomials provide a succinct, robust function for expressing glacial valley form that is not dependent on specification of the valley bottom. Polynomials were successfully applied to most non-stepped, symmetrical valley sections. The excellent fit of these models in the two Bear Valley transects developed in easily worked materials suggests that the polynomial form can be an appropriate model of glacially eroded morphology when structural controls are not dominant. These models also allow the partitioning of some stepped profiles into their component parts, and this technique successfully isolated inner gorges 200 and 300 m deep in Tenaya Canyon and Yuba Gorge, respectively.

Mathematical manipulation of polynomial coefficients provides quantitative information on quadratic valley sections including slopes, elevations, midpoint location, top width, depths, area, boundary length and shape. Testing with large numbers of glaciated valley cross-sections is needed to validate the findings of this report and to elucidate the environmental relationships between polynomial parameters and valley genesis. Through this endeavour, morphological features can be quantified and process-response relationships may emerge.

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